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A Method for Studying Spatial patterns: the Net-Function Interpolation

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Abstract

Interpolation is essential to most studies of spatial patterns in ecology. However, there has been a lack of quantitative techniques for interpolation of spatial data. This paper introduces a new method for studying spatial patterns, the two-dimensional net-function interpolation. The method can be used to interpolate unmeasured sample locations based on known values at nearby grid points. Specific examples from ecological studies in the Inner Mongolia Grassland, China are discussed to illustrate the use of the method. A brief comparison between the net-function interpolation and kriging is also made. The method may provide a useful tool assisting collection of field data of spatial patterning of physical and biological entities, and would allow ecologists to obtain vegetaion, soil and other ecological maps based on field observations with greatly reduced effort and over relatively large extent. Beyond that, the net-function technique seems to have potential for other fields (e.g., mining, forestry, meteorology, and structural geology) in which spatial interpolation plays a crucial role.

I. Introduction

The mosaic pattern and its multiplicity in spatial and temporal scales for a variety of ecological systems have long been noticed and studied by plant community ecologists (Watt, 1925, 1947; MacFadyen, 1950; Greig-Smith, 1952, 1983; Kershaw, 1957, 1960; Anderson, 1967, 1971; Pielou, 1977; Whittaker and Levin, 1977; Kershaw and Looney, 1985; C. Yang et al., 1984; Yang and Bao, 1986; Yang, 1988). The acutely increasing awareness and study of problems of scale and pattern in recent ecology (see Levin, 1992; Moloney et al., 1992; O'Neill et al., 1991 and Turner and Gardner, 1991 for recent reviews), spurred in particular by the development of landscape ecology, have renewed and greatly expanded the earlier interest conspicuously exemplified in plant community ecology. While the Greig-Smith/Kershaw blocking methodology predominated the early studies of patchy structure of plant communities, a number of relatively new techniques have been proposed and increasingly used for detection and analysis of spatial patterning across scales of landscapes. These tools include trend surface analysis, spline interpolation, autocorrelation, spectral analysis, fractals, semi-variograms, variance staircase, and moving window analysis (see Mucina et al., 1988; Levin, 1991; Turner and Gardner, 1991 and references therein). Each of the techniques, with its own advantages and disadvantages, is thus only suitable for certain particular aspects of spatial problems.

Spatial interpolation is essential to most ecological field studies; while taking average values for particular variable such as population densities, community biomass, or process rates over a spatial extent of landscape or an experimental plot, one in fact implicitly interpolates values for all unsampled points in space (Robertson, 1987). Interpolation usually becomes crucial and imperative especially in the field ecological investigation of spatial patterns, where missing data between observed sample points, due to particular field conditions, logistics, and other experimental constraints, are often encountered. With the renewed emphasis on spatial pattern and its heterogeneity and variability, a geostatistical method, kriging, has recently been used for spatial interpolation in ecological studies (e.g., Robertson, 1987; Robertson et al., 1988). Based on rather different mathematical theory, the net-function interpolation method may also be used in studies of spatial patterns and yet, the method is virtually unknown to the ecologist. In this paper, I will introduce the basics of the method, illustrate its application through ecological examples, and also make a brief comparison between the net-function interpolation and kriging.

II. The Two-Dimensional Net-Function Interpolation Method

The net-function interpolation method was developed by Peizhang Qiu and his associates at Inner Mongolia University (Qiu, 1978), adopting a mesh-generating numerical approach (Cook, 1974). The method allows one to estimate the functional values between lattice points according to those on the lattice points, and the procedure can be conveniently carried out numerically. Although the method can be used to interpolate functional values in an n-dimensional space (see Appendix), I will limit the discussion to the discrete, two-dimensional net-function interpolation case. The definition of net-function and derivation of the interpolation function are given in the Appendix.

The direct goal of the two-dimensional net-function interpolation is to obtain estimates of the internal unknown values based on the known on the boundary of a two-dimensional region. Figure 1 illustrates the spatial relationship among the net points at which the value is known or to be predicted. Given the z values at the net points along the four borders of the rectangular region $R^{(2)}$, the z value for any internal net point (x, y) can be calculated from the net-function interpolation equation (Qiu, 1978):

$$
\begin{aligned}
F(x, y) = {} & \frac{y - y_1}{y_0 - y_1} f(x, y_0) + \frac{y - y_0}{y_1 - y_0} f(x, y_1) + \frac{x - x_1}{x_0 - x_1} f(x_0, y) \\
& + \frac{x - x_0}{x_1 - x_0} f(x_1, y) - \frac{x - x_1}{x_0 - x_1} \left[\frac{y - y_1}{y_0 - y_1} f(x_0, y_0) + \frac{y - y_0}{y_1 - y_0} f(x_0, y_1)\right] \\
& - \frac{x - x_0}{x_1 - x_0} \left[\frac{y - y_1}{y_0 - y_1} f(x_1, y_0) + \frac{y - y_0}{y_1 - y_0} f(x_1, y_1)\right] \qquad (1)
\end{aligned}
$$

Alternatively, according to data availability and the researcher's preference, an area-based interpolation formula may be employed in place of the net point-based formula (eq. 1):

$$
\begin{aligned}
F(x, y) = {} & \frac{1}{A} \{ (A_3 + A_4) f(x, y_0) + (A_1 + A_2) f(x, y_1) + (A_2 + A_3) f(x_0, y) \\
& + (A_1 + A_4) f(x_1, y) - [A_3 f(x_0, y_0) + A_2 f(x_0, y_1) \\
& + A_4 f(x_1, y_0) + A_1 f(x_1, y_1)] \} \qquad (2)
\end{aligned}
$$

where A_1, A_2, A_3 and A_4 are the partial areas of the rectangular region $R^{(2)}$ with a total area of A, divided into the four parts by two lines perpendicular to each other and both passing point (x, y) (see Fig. 2).

Thus, eight known functional values on the boundary of $R^{(2)}$ are required to interpolate one unknown net point using the interpolation formula (Figs. 1 and 2). Four of the eight functional values correspond to the four corner positions of the rectangle $R^{(2)}$ on the XOY plane [i.e., $f(x_0, y_0)$, $f(x_0, y_1)$, $f(x_1, y_0)$, and $f(x_1, y_1)$], whereas the other four have the same coordinates, either on x or y axis, as does the inside point to be estimated [i.e., $f(x, y_0)$, $f(x, y_1)$, $f(x_0, y)$, and $f(x_1, y)$]. For example, let's consider a function f(x, y) of certain form defined over $R^{(2)}$ which is a unit square (say $0 \leq x \leq 1$ and $0 \leq y \leq 1$). When $f(x, y) = x^3 + xy^2$, at the point (3, 3) in $R^{(2)}$ the estimated value from the net-function interpolation formula [i.e., F(3, 3)] is exactly equal to the actual value [i.e., f(3, 3)] which is 54. When $f(x, y) = xy^3 + 4x^2 + 4xy + 6y^2$, the estimate F(0.1, 0.5) is 1.7525 while the actual value f(0.1, 0.5) is 1.753. The relative error [(f - F) / f] is 0.029%. When $f(x, y) = e^x + xy + x^2y^2$, F(0.1, 0.5) is 1.1352 and f(0.1, 0.5) is 1.1577, resulting in a relative error of 1.94%.

III. Use of the Net-Function Interpolation Method in a Multiple-Cell Grid

Because the variation of a variable tends to increase with spatial extent, to obtain reliable estimates using the two-dimensional net-function method may require interpolating over a nested network or grid with proper resolution (or grain size; see Fig. 3). This is especially true when small-scale patchiness or patterning is to be detected. In this case, the net-function interpolation formula may be repeated systematically over a regular-sized grid of, maybe, a large number of grid cells (Fig. 3). The number of grid points to be measured and to be estimated using the two-dimensional interpolation method can be computed according to the following formulas:

$$N_T = (m\,k + 1)\,(n\,k + 1) \tag{3}$$

$$N_E = m\;n\,(k - 1)^2 \tag{4}$$

$$N_M = N_T - N_E = (m\,k + 1)\,(n\,k + 1) - m\;n\,(k - 1)^2 \tag{5}$$

where N_T is the total number of grid points, N_E is the number of points to be calculated, N_M is the number of points to be actually measured, respective m and n are the number of grid cells along two neighboring sides of the rectangular transect (m = n for a square transect), and k is the number of subcells across a side of a grid cell.

In order to interpolate the N_E grid points simultaneously the basic formula of net-function interpolation (eq. 1) may be used to give rise to the following algorithm:

$$a_{ki+1-s,\, kj+1-r} = \frac{r}{k}\, a_{ki+1-s,\, kj+1-k} + \frac{(k-r)}{k}\, a_{ki+1-s,\, kj+1} + \frac{s}{k}\, a_{ki+1-k,\, kj+1-r}$$

$$+ \frac{(k-s)}{k} a_{ki+1, kj+1-r} - \frac{s}{k} \left[\frac{r}{k} a_{ki+1-k, kj+1-k} + \frac{(k-r)}{k} a_{ki+1-k, ki+1} \right]$$

$$- \frac{(k-s)}{k} \left[\frac{r}{k} a_{ki+1, ki+1-k} + \frac{(k-r)}{k} a_{ki+1, kj+1} \right] \qquad (6)$$

$$(r, s = 1, 2, ..., k - 1; \ i = 1, 2, .., N/D; \ j = 1, 2, ..., M/D)$$

where M and N are the side length of the transect, and D is the side length of a grid cell. The known values make up the following matrices:

$$[a_{ki+1, 1}, \ a_{ki+1, 2}, \ ..., a_{ki+1, m}] \qquad (i = 1, 2, ..., N/D)$$

and

$$\begin{bmatrix} a_{1, k_{j+1}} \\ a_{2, k_{j+1}} \\ . \\ . \\ . \\ a_{n, k_{j+1}} \end{bmatrix} \qquad (j = 1, 2, ..., M/D).$$

When the number of grid points is large for real problems, the use of computer becomes imperative for such computation (a computer program in C to implement the net-function interpolation for a given grid transect is available from the author upon request).

IV. Application of the Interpolation Method: Ecological Examples

The two-dimensional net-function interpolation method may be applied to the study of spatial distribution patterns of physical or biological entities or properties over a geographical region. The method was successfully applied in geophysical prospecting and drilling in China (Qiu, 1978) and in the study of spatial patterning of several plant species in an Aneurolepidium chinensis steppe community of the Inner Mongolia Grassland (C. Yang et al., 1984; Z. Yang et al., 1984; Yang and Bao, 1986; Yang, 1988). Here I will illustrate the use of the method in ecology based on the study of spatial patterning of plant communities by a group of scientists in Inner Mongolia University, Huhhot, China (see C Yang et al., 1984; Z. Yang et al., 1984).

The sampling work was conducted at the Inner Mongolia Grassland Ecosystem Research Station, Chinese Academy of Sciences, which is located in the Xilin River Basin (see Wu and Loucks, 1991 for a description of the physical environment and vegetation of this region). Five 50m X 25m transects were placed in a steppe community dominated by Caragana microphylla, Aneurolepidium chinense, Stipa grandis, and some other bunch grasses. Each transect was divided into 1,250 1m X

1m grid cells and each of them was further subdivided into 25 0.2m X 0.2m subcells (Fig. 3). Along the boundaries of 1m X 1m grid cell, the number of individuals of different species within 1cm from the line were counted at an interval of 0.2 m. The procedure proceeded from left to right for rows and from top to bottom for columns, and the values were assigned to the first-encountered grid points (Fig. 3). Bunch grasses were counted based on the number of clusters, whereas rhizome grasses counted as the number of above-ground stems (C. Yang et al., 1984).

In the study by Yang et al. (1986), the number of total grid points for each transect was 31,626, among which 11,626 were measured in the field and 20,000 were interpolated. The total number of individuals of each species could be estimated by using the interpolation method. For the purpose of studying the patch area and its distribution and also simplifying the computation process, the original abundance data were translated into presence/absence data (1's and 0's, see C. Yang et al., 1984). Each grid point represented an area of 0.04 m^2. After all the values of internal grid points were computed, a threshold level was used to convert the estimated values into one/zero series. The threshold was chosen so that the ratio of presence to absence from the estimated data set was equal or close enough to that from the measured data set.

Cover percentage of each population in the transect was estimated from the ratio of the grid points with positive and zero values following Monte Carlo methodology. From the interpolated maps using the net-function method, C. Yang et al. (1984) found that most plant populations under study were of aggregated distribution, with patches of more than one single spatial scale. This was largely in agreement with early studies using the Greig-Smith - Kershaw blocking method (see Yang, 1983; Yang and Bao, 1986). The average and maximum sizes of patches for 13 species were computed, and two-dimensional population distribution maps for each species were made to visualize the characteristics of spatial patterning within and between species. Moreover, single species distribution maps were overlaid so that the mosaic structure of the plant community was clearly manifested graphically.

Using the net-function interpolation method, Z. Yang et al. (1984) studied a random map of different-sized geometric shapes and a make-up distribution map of plant populations which mimicked a typical pattern as would be found in the steppe. Their results showed that the relative error (estimated versus actually measured patch coverage) was usually below 10% when the average area of smallest patches of interest was larger than the area of the grid cell. As expected, the estimation error would increase erratically once the average patch size was much smaller than the grid cell size.

V. Comparison between Kriging and the Net-Function Interpolation

"Kriging" conveys the same meaning as "optimal prediction" or "optimally predicting", referring to making inferences on unobserved values of a random process from observed at known spatial locations (Cressie, 1991). As a technique, it has been most extensively used in the mining industry for predicting ore values. Kriging, or geostatistics in general, recognizes that the set of random variables in question is characterized by a variance structure in which observations are not independent, but spatially correlated. An important basis for kriging is the regionalized variable theory that observations taken at short distance from each other are more alike (i.e., lower variances) than those taken further apart. Kriging allows for optimized estimation of such "regionalized variables" at unsampled locations based on values for nearby sample points weighted by distance and the degree of spatial autocorrelation. In essence, kriging procedure involves two steps: determining the spatial variance structure of the data set (i.e., the degree of spatial autocorrelation among the measured data points) and interpolating unmeasured values between sampled points (i.e., kriging) from known variance (see Journel and Huijbregts, 1978; Whitten, 1981; Robertson, 1987; Cressie, 1991).

Autocorrelation is evaluated by calculating the semi-variogram that defines the variance between observations at a distance h from another following the equation:

$$g(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [\, z(x_{i+h}) - z(x_i) \,]^2 \tag{7}$$

where $z(x_i)$ is the measured value at point x_i, $z(x_{i+h})$ is the sample value at point x_{i+h}, and $N(h)$ is the total number of pairs of sample points for a given interval (i.e., sample spacing), h. Semi-variogram parameters then can be used for kriging the sample values at unsampled locations through different or non-linear estimation algorithms (see Journel and Huijbregts, 1978; Journel, 1989; Cressie, 1991). These include ordinary kriging (punctual kriging at a point and block kriging for areas) and universal kriging (with trended data, i.e., the mean of the dependent variable changes across the study area).

Both kriging and the net-function interpolation methods assume that spatial variability at local scales is small and take into consideration the spatial relationship between sampled points and the spatial relationship between sampled points and the estimation point. The assumption seems empirically justified. Regionalization of physical and biotic properties may be expected from the nature of the geomorphological, pedogenic, and biological processes that influence them. As ecologists have long recognized, such processes and thus properties typically exhibit gradual rather than abrupt changes within short distance and across landscapes. However, the two methods are fundamentally different in that kriging is a probabilistic approach, whereas the net-function interpolation is deterministic without explicit consideration of spatial autocorrelation.

The net-function interpolation may not be able to provide constantly unbiased estimates for the points interpolated if the grid cell is too large. This is also likely to be true for kriging when h is becomes too large. In contrast with the net-function method, kriging permits statements of confidence levels to be associated with predictions. In addition, kriging can be used for data sets with irregularly-spaced sample points, although the construction of semi-variogram in such cases becomes complicated and sometimes introduces unacceptable bias (Whitten and Koelling, 1973; Whitten, 1981). The net-function method, on the other hand, appears to be undesirable for irregularly-gridded data. When anisotropy is taken into account with the universal kriging algorithm, kriging may become rather complex and computationally demanding (Whitten, 1981). It seems that the net-function method is numerically much simpler and the computational requirement increases only modestly with increasing grid size (e.g., C. Yang, 1984; Z. Yang, 1984).

VI. Discussion

The net-function interpolation method is potentially useful for detection and analysis of scales and patterns in ecology. However, the method has not, in effect, been used by ecologists. The method may prove useful especially in collecting field data on spatial patterning of physical and biological entities, which would allow the ecologist to afford to obtain spatial maps based on field observations with greatly reduced effort and over relatively large extent. The traditional and somewhat standard pattern analysis method in plant community ecology, the Greig-Smith - Kershaw technique would only be suitable and practical in rather small extent (e.g., Yang, 1983; C. Yang et al., 1984). Development of systematic and versatile computer programs with output formats suitable for pattern analysis will facilitate the application of the method. Also, the combination of this method with other pattern analysis techniques seems promising in future ecological studies. Although the method is discussed primarily in the ecological context in this paper, it should have potential in such fields as mining, forestry, meteorology, structural geology, or any others where spatial interpolation is of great importance.

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Appendix

Derivation of the (m) net-function interpolation function

An (m) net-function is defined as follows. Let $R^{(n)}$ be a rectangular body in an n-dimensional space ($n > 1$),

$$R^{(n)} = \{ (x_1, x_2, ..., x_n): x_i^{(0)} \leq x_i \leq x_i^{(m)}, 1 \leq i \leq n \} \tag{8}$$

which is divided by $n(m+1)^{n-1}$ straight lines into a meshwork. They all together form a manifold (m) mesh in $R^{(n)}$, and a function defined on this (m) mesh

$$f(x_1, x_2, ..., x_n) \qquad (\; R^{(n)} \;)$$

is called the (m) net-function with regard to $R^{(n)}$ (Qiu, 1978). When $n = 1$, $R^{(1)}$ becomes a linear segment (i.e., $x_1^{(0)} \leq x_1 \leq x_1^{(m)}$), and the (m) mesh is reduced to m+1 points [i.e., $x_1^{(di)}$, where di = 0, 1, ..., m] over which the (m) net-function $f(x_1)$ is defined.

One can construct a function $F(x_1, x_2, ..., x_n)$ from the (m) net-function (Qiu, 1978):

$$F(x_1, x_2, ..., x_n) \;\; ^{\circ} \quad \underset{1 \leq i_1 < i_2 < ... < i_{n-1} \leq n}{\bullet} \; L(x_{i_1})L(x_{i_2})...L(x_{i_{n-1}}) \, f(x_1,x_2,...,x_n) \; -$$

$$(n - 1) \; L(x_1)L(x_2)...L(x_n) \; f(x_1, x_2, ..., x_n) \tag{9}$$

where L(xj) is the (m) interpolation Lagrangian operator. Qiu (1978) further derived the (m) interpolation function:

$$F(x_1, x_2, ..., x_n) = \underset{j=1}{\overset{n}{\bullet}} \; \underset{0 \leq l_t \leq m}{\bullet} \; (x_1)^{l_1}(x_2)^{l_2}...(x_{j-1})^{l_{j-1}}(x_{j+1})^{l_{j+1}} ...(x_n)^{l_n} \; j(x_j) \tag{10}$$

where $F(x_1, x_2, ..., x_n)$ is defined on the closed region $x_j^{(0)} \leq x_j \leq x_j^{(m)}$.

The (m) interpolation function may be discrete. For example, let's consider the simple yet rather interesting case where n = 2 and m = 1. Suppose that a two-variable function [$z = f(x,y)$] is defined over a rectangular region $R^{(2)}$: $x_0 \le x \le x_1$, $y_0 \le y \le y_1$. That is, a (curved) surface is defined over a rectangle, bounded by four sides (linear boundaries): p_0, p_1, q_0, and q_1 (Fig. 4). If $R^{(2)}$ is horizontally placed, the functions for the four spatial curves are: $z = f(x_0, y)$, $z = f(x_1, y)$, $z = f(x, y_0)$, and $z = f(x, y_1)$, respectively.

Now let's create a new surface S': $z = F(x, y)$ to approximate the original surface S: $z = f(x, y)$. The new surface S' will have the same boundaries as S does, i.e.,

$$F(x_i, y) = f(x_i, y)$$

and $$F(x, y_j) = f(x, y_j) \qquad (i, j = 0, 1)$$

The procedures to create such a surface S' are as follows. First, construct a surface, S_1', by drawing straight line segments between q_0 and q_1 connecting all the points with the same x coordinates. The equation for the resulting surface is

$$S_1': \qquad F_1(x, y) = \frac{y - y_1}{y_0 - y_1} f(x, y_0) + \frac{y - y_0}{y_1 - y_0} f(x, y_1) \tag{11}$$

Alternatively, one may get

$$F_1(x, y) = \frac{x - x_1}{x_0 - x_1} f(x_0, y) + \frac{x - x_0}{x_1 - x_0} f(x_1, y) \tag{12}$$

Surface S_1' has two sides (i.e., q_0 and q_1) identical to S, with the other two (p_0^* and p_1^*) being different: $z = F_1(x_0, y)$ and $z = F_1(x_1, y)$, respectively.

Second, similarly construct a surface, S_2', by drawing straight lines between p_0 and p_1, which is described by the following equation:

$$S_2': \qquad F_2(x, y) = \frac{x - x_1}{x_0 - x_1} f(x_0, y) + \frac{x - x_0}{x_1 - x_0} f(x_1, y) \tag{13}$$

Alternatively,

$$F_2(x, y) = \frac{y - y_1}{y_0 - y_1} f(x, y_0) + \frac{y - y_0}{y_1 - y_0} f(x, y_1) \tag{14}$$

S_2' shares two of the four sides with S (i.e., p_0 and p_1) and the other two, q_0^* and q_1^*, are $z = F_2(x, y_0)$ and $z = F_2(x, y_1)$, respectively.

Third, construct an another surface, S_3', by connecting all pairs of points on p_0^* and p_1^* with the same x coordinates, resulting in

$$S_3': \qquad F_3(x, y) = \frac{x - x_1}{x_0 - x_1} \; F_1(x_0, y) + \frac{x - x_0}{x_1 - x_0} \; F_1(x_1, y) \tag{15}$$

Alternatively,

$$F_3(x, y) = \frac{y - y_1}{y_0 - y_1} \; F_1(x, y_0) + \frac{y - y}{y_1 - y_0} \; F_1(x, y_1) \tag{16}$$

The four sides of surface S_3' are p_0^*, p_1^*, q_0^*, and q_1^*, respectively. All the surfaces we have created, S_1', S_2' and S_3', are defined over the same region, $R^{(2)}$.

Lastly, obtain S' by combining S_1', S_2', and S_3', i.e.,

$$F(x, y) \;{}^{\circ}\; F_1(x, y) + F_2(x, y) - F_3(x, y) \tag{17}$$

Function F(x, y) is the (m=1) net-function interpolation formula for the spatial surface, z = f(x, y).

According to equations (11) - (17), therefore, we have the two-dimensional net-function interpolation formula (eq. 1):

$$\begin{aligned} F(x, y) \; = \; & \frac{y - y_1}{y_0 - y_1} \; f(x, y_0) + \frac{y - y_0}{y_1 - y_0} \; f(x, y_1) + \frac{x - x_1}{x_0 - x_1} \; f(x_0, y) \\ & + \frac{x - x_0}{x_1 - x_0} \; f(x_1, y) - \frac{x - x_1}{x_0 - x_1} \; [\frac{y - y_1}{y_0 - y_1} \; f(x_0, y_0) + \frac{y - y_0}{y_1 - y_0} \; f(x_0, y_1)] \\ & - \frac{x - x_0}{x_1 - x_0} \; [\frac{y - y_1}{y_0 - y_1} \; f(x_1, y_0) + \frac{y - y_0}{y_1 - y_0} \; f(x_1, y_1)] \end{aligned}$$

The absolute error, when F(x, y) is used to approximate f(x, y), is given by

$$g(x, y) \; = \; \frac{1}{4} \frac{\partial^4 f(x,h)}{\partial x^2 \partial y^2} (x - x_0)(x - x_1)(y - y_0)(y - y_1) \tag{18}$$

where x and h are dependent on the grid points to be calculated (x, y) and also satisfy the relationships: $x_0 \leq x \leq x_1$ and $y_0 \leq h \leq y_1$ (cf. Qiu, 1978).

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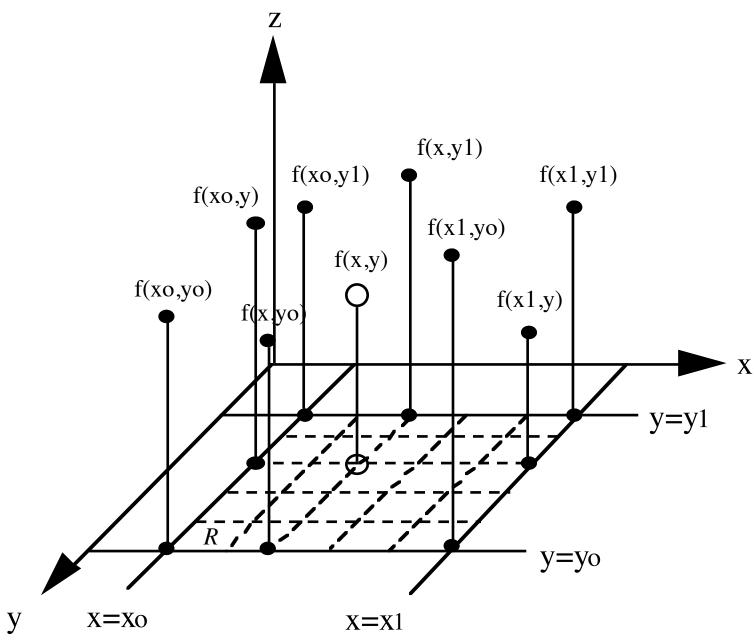

Figure 1. Schematic presentation of the two-dimensional net function interpolation. The z value of any internal net point (x,y) is estimated from the eight z values along the boundary of the region, R.

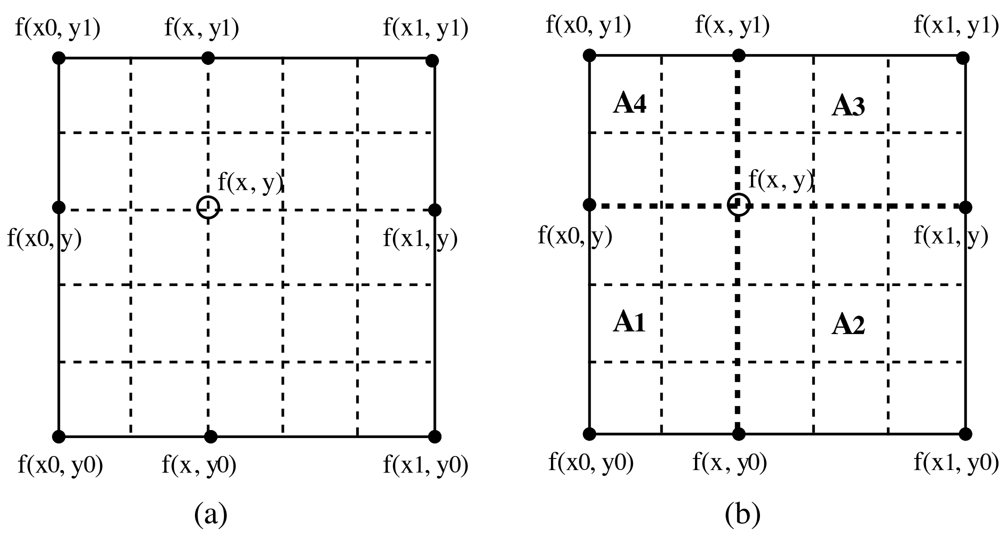

Figure 2. Illustration of the relationship between the point-based and area-based interpolation formulation schemes: (a) point-based and (b) area-based.

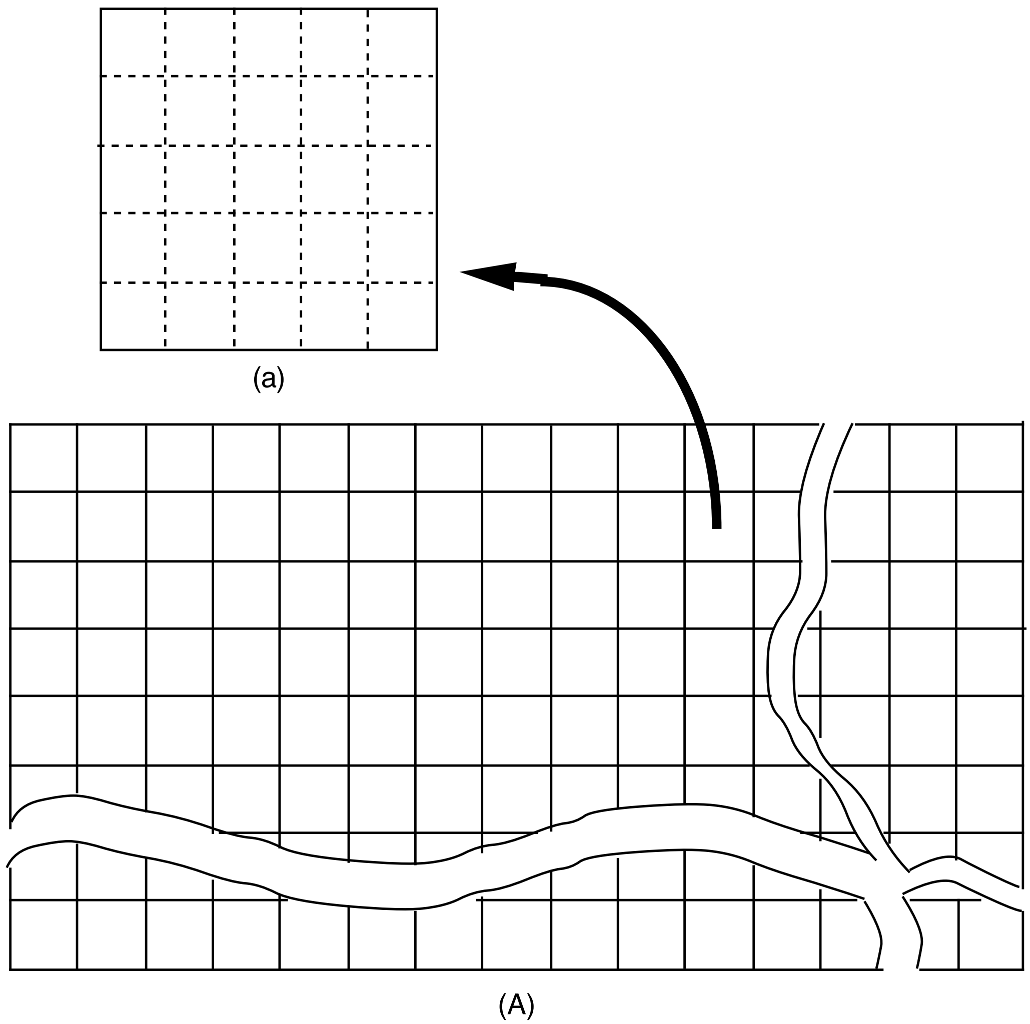

Figure 3. A grid of m X n cells (A), each of which is further divided into k X k subcells (a). In the grassland example, the grid is a 50m X 25m transect with 1,250 1m X 1m grid cells, and each grid cell is further divided into 25 20cm X 20cm subcells.

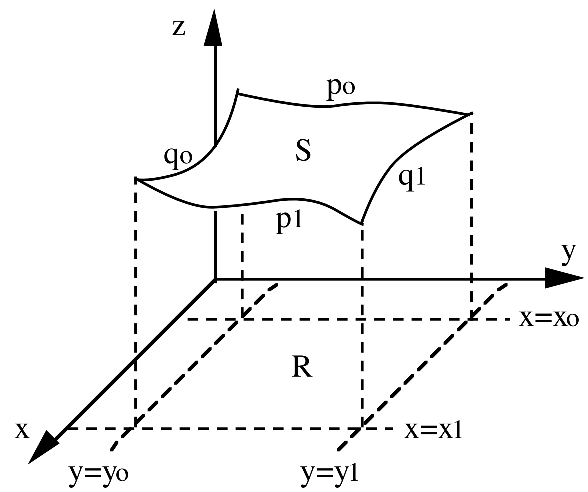

Figure 4. Illustration of a surface S defined over a rectangular region $R^{(2)}$.