NOTES ON CHAOS

1. SOME CONCEPTS

- Definition: The simplest and most intuitive definition of chaos is the extreme sensitivity of system dynamics to its initial conditions (Hastings et al. 1993).

- Chaotic dynamics stem from deterministic mechanisms, but they look very similar to random fluctuations in appearance.

- Among the earliest studies on chaos was the one by May (1974) of equilibrium, limit cycles and chaotic behavior in biological Populations with nonoverlapping Generations.

- Haefner (1996): "Chaos is a mathematical property of the time domain solutions of a set of equations and the parameters. Only nonlinear equations possess this property, so the study of chaos is a subset of nonlinear dynamics." While it's well known that simple nonlinear difference equations may generate chaos, such behavior may also arise in continuous systems with at least three state variables.

- Berryman and Millstein (1989): Chaotic dynamics may occur in systems with both positive and negative feedback loops when these systems are dominated much of the time by positive feedback growth processes. "Although all ecological systems contain the seeds of chaos (positive feedback), empirical evidence and evolutionary/ecological reasoning support the view that ecosystems do not normally behave chaotically. However, be driven to chaos by human actions that increase growth rates or induce delays in the regulatory (negative feedback) processes."

- Wu and Loucks (1995, From balance-of-nature to hierarchical patch dynamics: a paradigm shift in ecology. Quarterly Review of Biology 70:439-466): "Nonlinear systems can exhibit an important threshold phenomenon called bifurcation, in which abrupt, discontinuous changes in system behavior occur as a result of certain parameters crossing an apparent boundary of the domains of attraction (Levin, 1979; Sharma and Dettmann, 1989). The dynamics of these systems seem to be determined around threshold boundaries of great importance to which "equilibrium" may be expressed. More than 20 years ago, Holling (1973) indicated that the emphasis of research should be put on the boundaries of the domains of attraction, rather than on equilibrium states. Since then, a number of studies based on chaos theory and catastrophe theory have focused on threshold phenomena, resulting in new perspectives in the dynamics of ecological systems (e.g., May, 1975, 1977, 1986; Schaffer and Kot, 1985; Loehle, 1989; Sugihara and May, 1990). For example, the emergence of chaos theory has made scientists acutely aware of the complex dynamics and unpredictability of nonlinear systems, as well as the importance of threshold phenomena in nature. In view of chaos theory, various kinds of nonlinearities in pattern and process relationships can make some ecological systems sensitive to small changes in their conditions and thus inherently less predictable, a view that is ostensibly consistent with the theory of dissipative structure. In addition, chaos theory suggests that determinism does not necessarily increase the degree of system stability, rather, it is a condition for the emergence of chaotic behavior. Clearly, these views are in sharp contrast with the predictions of the classical equilibrium paradigm. The implications of chaos theory for ecology seem to be enormous, but are yet to be fully explored (Schaffer and Kot, 1985; Sugihara and May, 1990)."

- Godfray and Grenfell (1993): "Chaos should not be viewed purely as a property of deterministic systems, but as extreme sensitivity to initial conditions that can also arise through the interaction of the deterministic and stochastic components of a system.

2. A COMPARISON BETWEEN CHAOS AND RANDOM NOISE

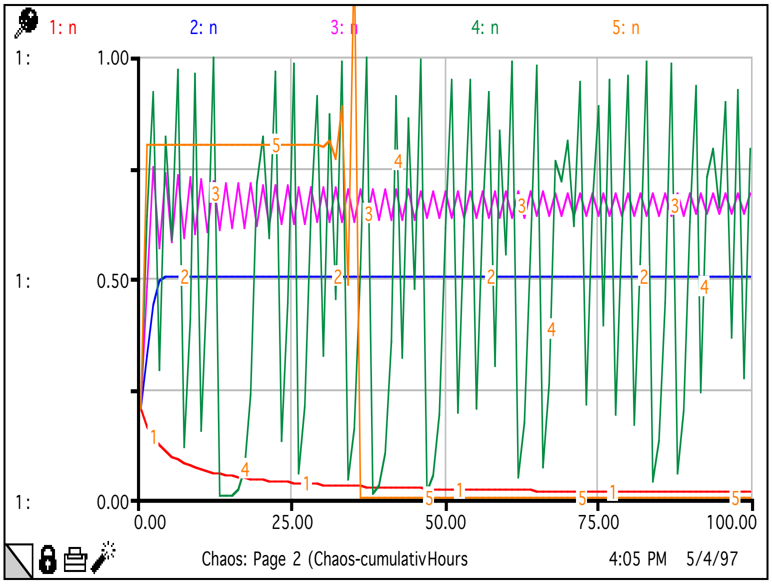
(1) A Modified Logistic Equation:

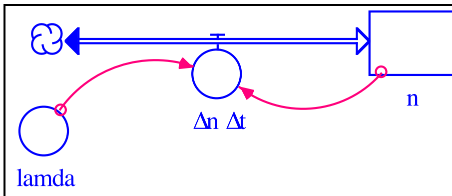
Let n be the relative population size with respect to the carrying capacity (i.e., N/K), a simple discrete model of population growth of the Logistic type can be written as:

$$n_{t+1} = \lambda\, n_t(1 - n_t)$$

where λ is the per capita growth rate of the population scaled by the carrying capacity.

When λ is 1, the population declines; When λ is 2, the population exhibits the typical logistic growth; when λ is 3, the population exhibits limit cycles; and when λ is equal to 4, chaos occur! This is also called Jenson chaos (Hannon and Ruth 1994).





n(t) = n(t - dt) + (Δn_Δt) * dt
INIT n = 0.2

INFLOWS:
Δn_Δt = lamda*n*(1 - n) - n
{where N is the population size, and r is a constant related to growth rate.}

lamda = 3.75
Last_N = DELAY(n,DT)

(2) The Discrete Version of the Logistic Model:

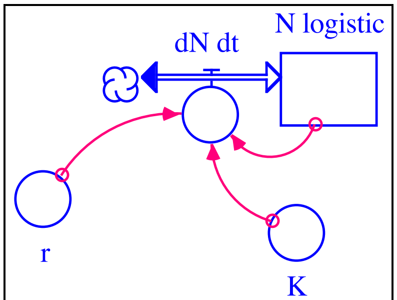
The most common continuous version of the Logistic model is

$$\frac{dN}{dt} = rN(1 - N/K)$$

The discrete version of the model is

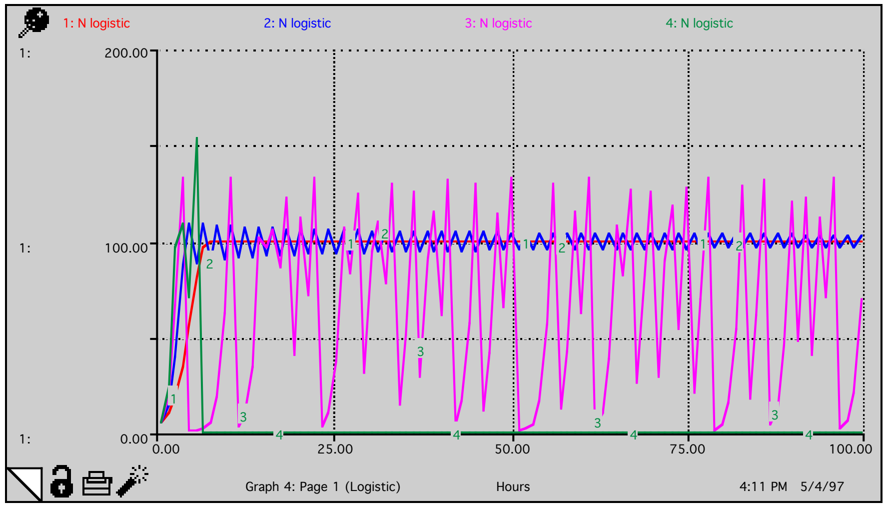
$$N_{t+1} = N_t + rN_t(1 - N_t/K)$$

This discrete Logistic model generates similar chaotic dynamics as the simple model discussed above, although the parameter values associated with chaos are different.



N_logistic(t) = N_logistic(t - dt) + (dN_dt) * dt
INIT N_logistic = 5

INFLOWS:
dN_dt = r * N_logistic * (1 - N_logistic/K)
K = 100
Last_N_logistic = DELAY(N_logistic, DT)
r = 4



3. REFERENCES

Allen, J. C. 1990. Factors Contributing to Chaos in Population Feedback Systems. Ecological Modelling, 51:281-298.

Berryman, A. A., and J. A. Millstein. 1989. Are Ecological Systems Chaotic-and if not, why not ? TREE, 4:26-28.

Cushing, J. M., B.Dennis, R. Desharnais, and R.F.Costantino. 1996. An interdisciplinary approach to understanding nonlinear ecological dynamics. Ecological Modelling, 92:111-119.

Godfray, H. C. J. and B.T. Grenfell. 1993. The continuing quest for chaos. TREE, 8:43-44.

Hastings, A., C.L.Hom, S.Ellner, P.Turchin, and H.C.J.Godfray. 1993. Chaos in Ecology: Is Mother Nature a Strange Attractor? Annu.Rev.Ecol.Syst., 24:1-33.

May, R. M. 1974. Biological Populations with Nonoverlapping Generations: Stable Points, Stable Cycles, and Chaos. Science, 186:645-647.

Schaffer, W. M., and M. Kot. 1985. Do Strange Attractors Govern Ecological Systems? BioScience, 35:342-350.

Schaffer, W. M. 1985. Order and Chaos in Ecological System. Ecology, 66(1):93-106.

Sugihara, G., B. Grenfell, and R. M. May. 1990. Distinguishing error from chaos in ecological time series. Phil. Trans. R. Soc. Lond., B 330:235-251.

Tilman, D., and D. Wedin. 1991. Oscillations and chaos in the dynamics of a perennial grass. Nature, 353:653-655.

Wilson, H. B., and D.A.Rand. 1993. Detecting chaos in a noisy time series. Proc. R. Soc. Lond. B:239-244.