1. **WHAT IS A SYSTEM?**
   - An organized collection of interrelated physical components characterized by a boundary and functional unity
   - A collection of “communicating” materials and processes that together perform some set of functions
   - An interlocking complex of processes characterized by many reciprocal cause-effect pathways
   - Any collection of interacting objects or processes
   - Any phenomenon, either structural or functional, having at least 2 separable components and some interaction between these components

2. **WHAT IS A MODEL?**
   - An **abstraction** or **simplification** of reality
   - A description of the essential elements of a problem and their relationships
   - A **reconstruction of nature** for the purpose of study (Levins 1968)
   - “A model is a caricature of nature ...... the simplest version of nature can be perturbed numerically with its responses being indications of the directions nature may take” (Scavia, Lang and Kitchell 1988).

   "Everything should be made as simple as possible, but not simpler."  
   --- Albert Einstein

   "While intelligent people can often simplify the complex, a fool is more likely to complicate the simple."  
   --- Swedish proverb

3. **TYPES OF MODELS**
   1) Physical vs. Symbolic (or abstract) Models
      - Physical: a scaled-down (or, less frequently, scaled-up) physical replicas of a real system
      - Symbolic: verbal, written language, or mathematical
   2) Dynamic vs. Static Models
      - Dynamic: with time varying variables
      - Static: time invariant
   3) Empirical (Correlative) vs. Mechanistic (Explanatory) Models
      - Empirical: without internal dynamics; descriptive; prediction
      - Mechanistic: with internal dynamics; explanatory; understanding/prediction
      - Whether a model is empirical or mechanistic is often relative, depending on the level of detail or the scale of observation.
   4) Deterministic vs. Stochastic Models
      - Deterministic: with no random variables; point estimation of parameters
      - Stochastic: with one or more random variables; estimation of parameter variations (e.g., mean, variance, distribution)
5) Simulation vs. Analytical Models
   o Analytic models: pencil and paper; solvable in closed form analytically; relatively complicated mathematics; greater mathematical power and usually higher generality
   o Simulation models: use of computers; relatively simpler mathematics; usually less generality and more realism

4. ORGANIZED VS. UNORGANIZED COMPLEXITIES AND MODELING APPROACHES:
   1) Small-number system:
      o Relatively few components, highly interrelated
      o Organized simplicity
      o Newtonian approach
   2) Medium-number
      o Intermediate number of components, closely interrelated
      o Organized complexity
      o Systems approach
   3) Large-number systems
      o Large number of components, loosely interrelated
      o Unorganized complexity
      o Statistical approach

❖ Rules of thumb for choosing methods of problem solving based on data availability and the level of understanding on the system:

<table>
<thead>
<tr>
<th>Amount of Data</th>
<th>Level of Understanding</th>
<th>Appropriate Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Many</td>
<td>Little</td>
<td>Statistics</td>
</tr>
<tr>
<td>Many</td>
<td>Good</td>
<td>Physics</td>
</tr>
<tr>
<td>Few</td>
<td>Little</td>
<td>System Analysis &amp; Simulation</td>
</tr>
<tr>
<td>Few</td>
<td>Good</td>
<td>System Analysis &amp; Simulation</td>
</tr>
</tbody>
</table>

5. WHY MODELS?

Models can be and have been used for several purposes other than prediction:
• Prediction
• Understanding
• Generate and test hypotheses
• Synthesis
• Identify areas of ignorance
• Serve as management tools
SYSTEMS THINKING AND SYSTEM MODELING

Systems Thinking

- Systems thinking is the art and science of making reliable inferences about system behavior by developing an increasingly deeper understanding of underlying structure and processes.

1) Structure-as-cause thinking (or the endogenous viewpoint):
   - Viewing the structure of a system as the cause of its behaviors, as opposed to seeing the behaviors as being imposed upon the system by outside agents.

2) Closed-loop thinking:
   - Structural elements of the system are arranged in closed-loops (feedback loops).
   - Emphasizing reciprocal causal relationships

3) Operational thinking:
   - What are closed-loops composed of? Primarily, stocks (state variables) and flows (rate variables).
   - System components form feedback loops which ultimately determine the behaviors of the system.

Systems Analysis

1) What Is Systems Analysis?
   - A methodology originally developed by the military to deal with the complex logistical problems during World War II.
   - Systems analysis consists of the determination of important system components, systems simulation, systems optimization, and systems measurement (Watt 1968).
   - The application of scientific method to complex problems, which is distinguished by the use of advanced mathematical and statistical techniques and by the use of computers (Dale 1970)

2) Systems Models and Systems Diagrams
   - Stocks (or state variables or accumulations) are the major concerns which change through time (e.g., population size, biomass, etc.).
   - Flows (or rate variables) represent flows of material or energy between state variables and characterize the rate of change of these state variables as a result of specific processes.
   - Information links between variables are established by auxiliary variables (converters).
Symbols used in systems modeling (Forrester diagram)

Four Basic Phases in Systems Analysis (or Systems Modeling)

Phase 1: Conceptual model formulation
- Define the problem
- Specify model objectives
- Delineate system boundaries
- Construct causal diagrams
- Understand feedback loop structure

Phase 2: Quantitative model specification
- Select general quantitative structure for the model
- Choose basic time step for simulations
- Identify functional forms of model equations
- Estimate parameters of model equations
- Code model equations for the computer
- Execute baseline simulation
- Present model equations
Phase 3: Model evaluation

- Components of model evaluation
- Model verification
- Model validation

Phase 4: Model application

- Develop and execute experimental design for simulations
- Analyze and interpret simulation results
- Examine additional types of management policies or environmental situations
- Communicate simulation results

* NOTE: The four phases are interactive, meaning that one often has to go back and forth among the four phases during model development.

System Dynamics and STELLA

1) System Dynamics (SD)

- Founded by Forrester and his associates at MIT in 1950s (Forrester, 1961, 1968)
- Holds that the macro behavior of a system is primarily determined by its internal micro structure
- Emphasizes the connections among the various parts that constitute a system and applies feedback principles to model and analyze dynamic problems
- Holds that the core of system structure is composed of feedback loops (positive and negative) that integrate the three fundamental constituents: state, rate, and information.
- DYNAMO – the computer language born with System Dynamics.
- Based on general systems theory, incorporates cybernetics and information theory, and has become a unique, powerful simulation modeling methodology

2) STELLA

- STELLA (acronym for Structural Thinking Experiential Learning Laboratory with Animation)
- Icon-oriented and self-debugging systems simulation package
- First released in August 1985 by High Performance Systems, Hanover, New Hampshire
- Designed to facilitate System Dynamics modeling and make it available for even those lacking computer experience and mathematical expertise
- Awarded as “the best published piece of work in the [SD] field between 1984 and 1989” (Forrester Award Committee)
Symbols used for structural diagrams in STELLA
I. WHY BUILDING BLOCKS?

Building blocks, or simulation modules are simple models that represent some basic system structures and dynamics. These modules are very important for understanding many fundamental processes common in biological, physical, and socioeconomic phenomena. One certainly needs to understand them well before attempting to deal with complex feedback systems. In the same time, the model building blocks demonstrate how these commonly occurring basic processes are represented in systems simulation, and often become convenient and effective for constructing complex systems models.

II. BUILDING BLOCKS FOR SYSTEMS MODELS

(I) FIRST-ORDER SYSTEMS

- First-order: One state variable (or 1 stock)
- Linear: No non-linear combinations of the state variable of any sort in the algebraic equation of rates.

1. Linear Change

   - One state variable.
   - One rate variable.
   - The rate of change is a constant, not dependent on the state variable. Thus, there is no feedback loop.

(1) Linear Growth

   - The rate of change is positive, and thus represented as an inflow.
   - Examples:
     - Increase in water level in a tank with a constant inflow of water into the tank.
     - Plant biomass sometimes is found to increase linearly with certain soil nutrients.

\[
X(t) = X(t - dt) + (\frac{dX}{dt}) * dt
\]

\[
IN\ INIT\ X = 0
\]

\[
IN\ INFLOWS:
\]

\[
\frac{dX}{dt} = 2
\]
(2) Linear Decline
- The rate of change is negative, and thus represented as an outflow.
- Examples:
  - Decrease in the amount of materials left in a stock when the materials are taken out of the stock in the same amount per unit of time.
  - Decrease in crop yield with declining annual precipitation sometimes exhibits linear pattern.

(3) Linear Growth and Decline
- Combination of the linear growth and decline module.
- One state variable.
- Either two uniflows (one inflow and one outflow) or one biflow (compare the structural diagrams below).
- The uniflow version gives more details, whereas the biflow version is more concise. The choice of the two forms is dependent on the degree of details is needed.
- Linear growth occurs when the total rate of change (inflow minus outflow) is positive, and linear decline does otherwise.
2. Exponential Growth and Decay

(1) Exponential Growth
- Also known as compounding process
- One state variable.
- One rate variable.
- One auxiliary variable.
- The rate of change is equal to a constant proportion of the state variable.
- Because the rate variable depends on the state variable itself, they form a positive feedback loop, in which the state variable and the rate of change reinforce each other, generating an accelerating, runaway behavior.
- The rate of change is always positive (i.e., the state variable increases monotonically), and is represented as an inflow.
- **Time Constant** $T_c$: The reciprocal of the constant $r$ in the exponential model, $T_c = 1/r$, with the dimension of units of time.
  - For exponential growth, $T_c$ is the time required for the state variable to become $e (= 2.7183)$ times of its current value. This can be seen from the following simple manipulations:
    \[ N = N_0 e^{rt}. \]
    When $t$ is equal to $T_c$, we have
    \[ N_{T_c} = N_0 e^{1/r} = N_0 e \]
    In general, if $t = nT_c$,
    \[ N_{nT_c} = N_0 e^{\frac{1}{T_c}} = N_0 e^n \]
- Note: For continuous systems, the time step, $\Delta t$, in simulation must be smaller than the smallest time constant in the model.
• **Doubling Time** $T_d$: The time required for the state variable in the exponential model to double in value.

To calculate the doubling time, let $N = 2N_0$ and let $t = T_d$.

Thus, from $N = N_0 e^{rt}$, we have:

$$2N_0 = N_0 e^{rT_d}$$

$$\ln 2 = rT_d$$

$$T_d = \frac{0.69}{r} \quad \text{or} \quad T_d = 0.69 \ T_c$$

Therefore, the **doubling time** for exponential growth is a constant. It is conversely proportional to the rate of change: doubling time = 70 / percent natural growth rate. This is where “the rule of 70” in population demography comes from.

• **Exponential growth** is a common type of dynamics that exists in all different disciplines. Any phenomena described by words such as “snowball effect”, “vicious circles”, “virtuous circles”, and “bandwagon effect” can be represented as a positive feedback loop structure.

Examples include: Savings increase in a bank account due to interest income; Population growth when resources are not limiting; Drug addiction; Arms race; etc.

• **Super-exponential** (or supra-exponential) growth:

  o In some positive feedback systems, doubling time decreases, rather than remain constant, as the state variable increases. This means that the rate of change is a nonlinear function of the state variable, e.g.:

  $$\frac{dN}{dt} = (rN)N = rN^2$$

  o The dynamics generated by this **nonlinear** system, with a rate of change faster than a fixed proportion of the state variable, is called superexponential growth.

  o The superexponential growth has the same feedback structure as the exponential.
(2) Exponential Decay

- Also known as *draining process*
- 1 state variable; 1 rate variable.
- The rate of change is equal to a constant proportion of the state variable, but in contrast with exponential growth it is negative.
- The rate variable depends on the state variable itself, and form a feedback loop between them. Because an increase in the value of the state variable increases the rate of change, but an increase in the rate decreases the value of the state variable, the feedback is **negative** (or goal-seeking).

- **Time constant** $T_c$: The reciprocal of the constant $r$ in the exponential model, $T_c = 1/r$, with the dimension of units of time.
  - For exponential decay, $T_c$ is the time required for the state variable to become $e^{-1}$ ($=0.368$) times of its current value, or the time required for 63% of the contents in the stock to vanish. This is also called the relaxation time, which is sometimes used as a measure of the speed with which a system is absorbing disturbances.
  - Mathematically, relaxation time can be derived as follows:
    \[
    N = N_0 e^{-rT_c},
    \]
    For a time period from $t=0$ to $t=T_c$,
    \[
    N_{T_c} = N_0 e^{-T_c/r} = e^{-1}N_0 = 0.368N_0,
    \]
    In general, when $t = nT_c$, we have
    \[
    N_{nT_c} = N_0 e^{-nT_c/r} = e^{-n}N_0 = 0.368^nN_0
    \]

- **Half-life** $T_h$: The time required for the state variable to reduce its current value by a half. It is an analog of the doubling time in exponential growth. It can be calculated as follows:
  \[
  \frac{1}{2}N_0 = N_0 e^{-rT_h},
  \]
  \[
  \ln \frac{1}{2} = -rT_h, \text{ or } \ln 2 = rT_h, \text{ thus,}
  \]
  \[
  T_h = \frac{0.69}{r} \quad \text{or} \quad T_h = 0.69T_c
  \]
  - Therefore, the half-life time for exponential decay is a constant (about 70% of the time constant), independent of the value of the state variable.
• Examples:
  o Draining water from a tank
  o Population-death rate process
  o Radioactive decay process

(3) Exponential Growth and Decay Combined
  o Simply a combination of exponential growth and exponential decay.
  o The behavior of the first-order linear system exhibits three different patterns:
    1) exponential growth when growth constant is larger than decay constant;
    2) exponential decay when growth constant is smaller than decay constant; and
    3) remaining unchanged when the two constants are equal.

3. Exponential Collapse
• Also known as accelerated decay, indicating that the rate of change gets faster as the level goes down.
• A simple exponential collapse model may consist of 1 state variable, 1 outflow, and 2 constants.
• A positive feedback: increasing level --> increasing rate of change --> increasing level --> ...
• A simple mathematical model for exponential collapse:
  \[
  \frac{dN}{dt} = -r(M - N)
  \]
  where \(N\) (< \(M\)) is the state variable, and \(r\) and \(M\) are two constants.

• Examples:
  o Population shrinking when smaller than MVP (minimum viable population).
  o Change in the interactive force
between molecules when water is heated up and boils.
- Decline of species diversity with increasing habitat fragmentation.
- Some other threshold phenomena in physical and biological processes may exhibit behavior that resembles exponential collapse.

4. Exponential Growth and Collapse Combined
- A simple exponential collapse consists of 1 state variable, 1 bi-directional flow, and two constants.
- This simple model can be mathematically expressed as:
  \[
  \frac{dN}{dt} = r(N - M)
  \]

The simple structure exhibits 3 different behavioral patterns:
- Exponential growth when \( N > M \);
- Exponential collapse when \( N < M \);
  and
- Remaining constant when \( N = M \).

Exponential Growth/Collapse in STELLA

- level(t) = level(t - dt) + (-outflow)*dt
- INIT level = 99
- OUTFLOWS:
  - outflow = collapse_constant*(M-level)
  - collapse_constant = 0.15
  - M = 100

Graph 1: Page 1 (Exponential Time) 3:06 PM 4/10/97
5. Goal-Seeking Behavior

(1) Simple Goal-Seeking
- A simple goal-seeking model may consist of 1 state variable, 1 biflow, and 2 constants.
- The state variable and the flow form a negative feedback.
- A simple goal-seeking model may take the form:
  \[ \frac{dL}{dt} = c(G - L) \]
  where \( L \) is the level, \( G \) is the goal, and \( c \) is a constant.
- Apparently, exponential decay is a special case of goal-seeking behavior, in which the goal is zero!

(2) S-Shaped Growth
- Also known as logistic growth or sigmoidal growth.
- It is a first-order nonlinear system (see the Rate-Level graphs).
- The state variable and the flow form a negative feedback that ultimately gives rise to the goal seeking behavior (see the Forrester diagram).
- It may take different forms (uniflow or biflow versions). [For example, in population regulation, crowding may affect only the per capita birth rate, or only the per capita death rate, or both.]

The behavior of this simple structure has three patterns:
  1) Increasing and approaching the goal if simulation starts with a value of the level smaller than the goal;
2) Decreasing and approaching the goal if simulation starts with a value of the level larger than the goal;
3) Remaining unchanged if simulation starts with the value of the level equal to the goal.

A familiar example of sigmoidal growth is the logistic equation:

\[
\frac{dN}{dt} = rN(1 - N/K)
\]

This is a first-order nonlinear differential equation. The following is an equivalent simple systems model.

\[
N(t) = N(t - dt) + (dN/dt) * dt
\]

INIT N = 2

INFLOWS:
\[dN/ dt = r*N*(K-N)/K\]
K = 100
r = 0.2

Dynamics from the Logistic Equation-Based Module